

Gauge invariant Lagrangian construction for massive bosonic mixed symmetry higher spin fields

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Abstract

We develop the BRST approach to gauge invariant Lagrangian construction for the massive mixed symmetry integer higher spin fields described by the rank-two Young tableaux in arbitrary dimensional Minkowski space. The theory is formulated in terms of auxiliary Fock space. No off-shell constraints on the fields and the gauge parameters are imposed. The approach under consideration automatically leads to a gauge invariant Lagrangian for massive theory with all appropriate Stückelberg fields. It is shown that all the restrictions defining an irreducible representation of the Poincare group arise from Lagrangian formulation as a consequence of the equations of motion and gauge transformations. As an example of the general procedure, we derive the gauge-invariant Lagrangian for massive rank-2 antisymmetric tensor field containing the complete set of auxiliary fields and gauge parameters.

1 Introduction

As known, in higher dimensions, $d > 5$, the totally symmetric tensor fields are not enough to cover all the irreducible representations of the Poincare group. We should take into account the fields with mixed symmetry of the indices as well. It is interesting that such mixed symmetry fields naturally arise in context of superstring theory. Therefore, a higher spin field theory in higher dimensions should describe a mixed symmetry fields dynamics. Approaches to Lagrangian formulation for the massless mixed symmetry fields in Minkowski space were developed in refs. [1]. Analogous problems in AdS space were carefully analyzed in [2], where the massless fields of generic symmetry type were studied and an example of the Lagrangian for field corresponding to the three-cell 'hook' diagram was constructed (see also [3]). Using dimensional reduction equations of motion in the Stückelberg formalism for the massive 'hook' representation were obtained in [4], aspects of Lagrangian formulation for massive higher spin mixed symmetry fields were considered in refs. [5].

In this note, we develop a general approach to gauge invariant Lagrangian formulation for any massive bosonic mixed symmetry tensor fields described by the rank-two Young tableaux

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in arbitrary dimensional Minkowsky space time. Our approach is based on BRST-BFV method which earlier was applied for Lagrangian construction for free totally symmetric massless and massive, bosonic and fermionic higher spin fields in flat and in AdS spaces [6, 7]¹. Also the aspects of cubic interaction for bosonic massless higher spin field were studied in this approach [9].

The paper is organized as follows. In section 2 we briefly consider a description of massive irreducible tensor representations of d -dimensional Poincare group in terms of Fock space. To obtain the irreducible tensors we should impose the specific restrictions on the fields. Then one introduces the standard creation and annihilation operators and rewrites the above restrictions as the operator constraints in Fock space. These operator constraints, together with operators obtained from the constraints by Hermitian conjugation, generate a closed algebra in terms of commutators. In section 3 we construct, following refs. [7] a new representation of this algebra. In section 4 we derive the BRST-BFV operator which then is used for constructing the Lagrangian and gauge transformations. In section 5 we prove that equations defining the irreducible representations are obtained from the found Lagrangian after partial gauge fixing. In section 6 we consider two examples. The first example shows that if we consider totally symmetric tensor field corresponding to 1-row Young tableau, then the method is reduced to the case which is already studied in [7]. The second example deals with rank-2 antisymmetric tensor field. We find the Lagrangian, the gauge transformations of the fields, and the gauge transformation of the gauge parameters. In section 7 we summarize the results and briefly discuss general mixed symmetric theory.

2 Irreducible mixed symmetric tensor representation of Poincare algebra

In this paper we are going to construct a Lagrangian for the massive tensor fields corresponding to a Young tableau with 2 rows ($s_1 \geq s_2$)

$$\Phi_{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}}(x) \longleftrightarrow \begin{array}{|c|c|c|c|c|c|} \hline \mu_1 & \mu_2 & \dots & \dots & \dots & \mu_{s_1} \\ \hline \nu_1 & \nu_2 & \dots & \nu_{s_2} & & \\ \hline \end{array}. \quad (1)$$

In the correspondence with given Young tableau, the tensor field is symmetric with respect to permutation of each type of the indices² $\Phi_{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}}(x) = \Phi_{(\mu_1 \dots \mu_{s_1}), (\nu_1 \dots \nu_{s_2})}(x)$ and that after symmetrization of all indices corresponding to the first row with one index corresponding to the second row the field vanishes

$$\Phi_{(\mu_1 \dots \mu_{s_1}, \nu_1) \dots \nu_{s_2}}(x) = 0. \quad (2)$$

We consider irreducibility condition under the mass stability subgroup of the Poincare group which results in the traceless condition³

$$\eta^{\mu_1 \mu_2} \Phi_{\mu_1 \mu_2 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}} = \eta^{\nu_1 \nu_2} \Phi_{\mu_1 \dots \mu_{s_1}, \nu_1 \nu_2 \dots \nu_{s_2}} = \eta^{\mu_1 \nu_2} \Phi_{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}} = 0. \quad (3)$$

For irreducibility under the Poincare group we impose the Klein-Gordon equation and the transversality conditions

$$(\partial^2 + m^2) \Phi_{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}}(x) = 0, \quad (4)$$

$$\partial^{\mu_1} \Phi_{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}}(x) = \partial^{\nu_1} \Phi_{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}}(x) = 0. \quad (5)$$

¹The massless higher spin mixed symmetry fields were analyzed with help of BRST-BFV method in refs. [8].

²The indices inside round brackets are to be symmetrized.

³we use the metric $\eta^{\mu\nu} = \text{diag}(+, -, -, \dots, -)$

To avoid the explicit manipulations with a big number of indices it is convenient to introduce Fock space generated by creation and annihilation operators

$$[a_i^\mu, a_j^{+\nu}] = -\eta^{\mu\nu} \delta_{ij}, \quad i, j = 1, 2. \quad (6)$$

The number of pairs of creation and annihilation operators one should introduce is determined by the number of rows in the Young tableau corresponding to the symmetry of the tensor field. Thus, unlike the totally symmetric field case [7] we introduce two pairs of such operators.

An arbitrary state vector in this Fock space has the form

$$|\Phi\rangle = \sum_{s_1=0}^{\infty} \sum_{s_2=0}^{\infty} \Phi_{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}}(x) a_1^{+\mu_1} \dots a_1^{+\mu_{s_1}} a_2^{+\nu_1} \dots a_2^{+\nu_{s_2}} |0\rangle. \quad (7)$$

One sees that the symmetry properties of the coefficient functions $\Phi_{(\mu_1 \dots \mu_{s_1}), (\nu_1 \dots \nu_{s_2})}(x)$ are stipulated by the symmetry properties of the product of the creation operators. To get another restrictions (2)–(5) on the coefficient functions we introduce the following operators

$$l_0 = -p^\mu p_\mu + m^2, \quad l_i = a_i^\mu p_\mu, \quad l_{ij} = \frac{1}{2} a_i^\mu a_{j\mu}, \quad g_{12} = -a_1^{+\mu} a_{2\mu} \quad (8)$$

where $p_\mu = -i\partial_\mu$. One can check that restrictions (2)–(5) are equivalent to

$$g_{12}|\Phi\rangle = 0, \quad l_{ij}|\Phi\rangle = 0, \quad l_0|\Phi\rangle = 0, \quad l_i|\Phi\rangle = 0 \quad (9)$$

respectively.

Our purpose is to construct a Lagrangian on the base of Hermitian BRST-BFV operator. Therefore the set of operators which is used for constructing the BRST-BFV operator must be closed under Hermitian conjugation and form a closed algebra. Thus, first, to form a set of operators which is closed under Hermitian conjugation we add to operators (8) their Hermitian conjugate

$$l_i^+ = a_i^{+\mu} p_\mu, \quad l_{ij}^+ = \frac{1}{2} a_i^{+\mu} a_{j\mu}^+, \quad g_{12}^+ = -a_2^{+\mu} a_{1\mu} \equiv g_{21} \quad (10)$$

and second, we add to the set of operators all the operators which are necessary to form a closed algebra

$$m^2, \quad g_{11} = -a_1^{+\mu} a_{1\mu} + \frac{d}{2}, \quad g_{22} = -a_2^{+\mu} a_{2\mu} + \frac{d}{2}. \quad (11)$$

Now we have the set of operators (8), (10), (11) which is closed under Hermitian conjugation and form an algebra⁴ which given in Table 1.

One can show that a straightforward use of the BRST-BFV construction as if all the operators were first class constraints doesn't lead to the proper equations (9) for any value of spin (see e.g. [7] for the cases of higher spin fields corresponding to one-row Young tableau). This happens because among the above Hermitian operators there are operators which are not constraints (m^2 , g_{11} , g_{22} in the case under consideration) and they produce more equations (in addition to (9)) for the physical field (7). Thus we must somehow get rid of these supplementary equations. A method of avoiding such supplementary equations consists of constructing new enlarged expressions for the operators of the algebra given in Table 1 so that the hermitian operators which are not constraints will be zero.

⁴It is worth noting that the same algebra of operators can be obtained by first quantization of a spinning particle model [10]. We are grateful to A. Waldron for drawing our attention on this work.

$[\downarrow, \rightarrow]$	m^2	l_0	l_k	l_k^+	l_{kl}	l_{kl}^+	g_{kl}
$m^2 = m^2$	0	0	0	0	0	0	0
$l_0 = -p^2 + m^2$	0	0	0	0	0	0	0
$l_i = p^\mu a_{i\mu}$	0	0	0	$\delta_{ik}(l_0 - m^2)$	0	$-\delta_{i(l} l_k^+$	$\delta_{i(k} l_l)$
$l_i^+ = p^\mu a_{i\mu}^+$	0	0	\dots	0	$\delta_{i(l} l_k)$	0	$-\delta_{i(l} l_k^+$
$l_{ij} = \frac{1}{2} a_{i\mu} a_j^\mu$	0	0	0	\dots	0	$\frac{1}{2}(\delta_{k(i} g_{j)l} + \delta_{l(i} g_{j)k})$	$2\delta_{k(i} l_{j)l}$
$l_{ij}^+ = \frac{1}{2} a_i^{\mu+} a_{j\mu}^+$	0	0	\dots	0	\dots	0	$2\delta_{l(i} l_{j)k}^+$
$g_{ij} = -a_{i\mu}^+ a_j^\mu + \frac{d}{2} \delta_{ij}$	0	0	\dots	\dots	\dots	\dots	$g_{il} \delta_{jk} - g_{kj} \delta_{il}$

Table 1: Algebra of the initial operators

3 New representation of the algebra

According to the method developed in our previous papers [7], in order to avoid the supplementary equations for the basic field one should construct new expressions for the operators of the algebra given in Table 1. These new expressions L_a are sums of the initial operators l_a and additional parts l'_a , that is $l_a \rightarrow L_a = l_a + l'_a$. The additional expressions are constructed from new (additional) creation and annihilation operators and parameters of the theory (the mass m in the theory under consideration) and so commute with the initial operators. The requirements on the additional parts are: 1) L_a must form an algebra $[L_a, L_b] \sim L_c$; 2) the additional parts corresponding to the operators which produce the supplementary equations on the basic field must either linearly contain arbitrary parameters whose values shall be defined later or give zero when they are added to the corresponding initial operators.

Since the initial operators commute with the additional parts $[l_a, l'_b] = 0$ and since the algebra of the initial operators is a Lie algebra then as was shown in [7] the additional parts must form the same algebra as the initial operators (see Table 1). Explicit expressions for the additional parts satisfying a given algebra can be constructed using the method adopted in [11], [7].

According to this method we introduce six pairs of bosonic creation and annihilation operators with the standard commutation relations

$$[b_{11}, b_{11}^+] = [b_{22}, b_{22}^+] = [b_{12}, b_{12}^+] = [b, b^+] = [b_1, b_1^+] = [b_2, b_2^+] = 1 \quad (12)$$

and then derive explicit expressions for the additional parts⁵

$$l'_0 = 0, \quad l'_i = m b_i, \quad l_i^+ = m b_i^+, \quad l_{ij}^+ = -\frac{1}{2} b_i^+ b_j^+ + b_{ij}^+, \quad m'^2 = -m^2, \quad (13)$$

⁵ There are differences from paper [8] because of different definitions of generators (8).

$$l'_{11} = (h_1 - N_G)b_{11} + \frac{1}{2}b^+b_{12} + (N_{11} + N_{12})b_{11} + \frac{1}{4}b_{22}^+b_{12}^2 - \frac{1}{2}b_1^2, \quad (14)$$

$$l'_{22} = (h_2 + N_G)b_{22} + \frac{1}{2}(h_1 - h_2 - N_G)bb_{12} + (N_{22} + N_{12})b_{22} + \frac{1}{4}b_{11}^+b_{12}^2 - \frac{1}{2}b_2^2, \quad (15)$$

$$2l'_{12} = (h_1 - h_2 - N_G)bb_{11} + b^+b_{22} + \frac{1}{2}(h_1 + h_2)b_{12} + \frac{1}{2}(2N_{11} + 2N_{22} + N_{12})b_{12} + 2b_{12}^+b_{11}b_{22} - b_1b_2, \quad (16)$$

$$g'_{11} = h_1 - N_G + 2N_{11} + N_{12} + b_1^+b_1 + \frac{1}{2}, \quad (17)$$

$$g'_{22} = h_2 + N_G + 2N_{22} + N_{12} + b_2^+b_2 + \frac{1}{2}, \quad (18)$$

$$g'_{12} = (h_1 - h_2 - N_G)b + b_{11}^\dagger b_{12} + 2b_{12}^\dagger b_{22} + b_1^+b_2, \quad (19)$$

$$g'_{21} = b^+ + b_{22}^\dagger b_{12} + 2b_{12}^\dagger b_{11} + b_2^+b_1. \quad (20)$$

where

$$N_{ij} = b_{ij}^+ b_{ij}, \quad N_G = b^+b. \quad (21)$$

The found additional parts possess all the necessary properties described at the beginning of the present section. The enlarged operators

$$L_0 = l_0 + l'_0, \quad L_i = l_i + l'_i, \quad L_{ij} = l_{ij} + l'_{ij}, \quad G_{ij} = g_{ij} + g'_{ij}, \quad (22)$$

$$M^2 = m^2 + m'^2, \quad L_i^+ = l_i^+ + l'^+, \quad L_{ij}^+ = l_{ij}^+ + l'^+, \quad G_{12}^+ \equiv G_{21} \quad (23)$$

form an algebra (which coincides with the algebra given in Table 1) and the additional parts g'_{11} , g'_{22} corresponding to the operators g_{11} , g_{22} which are not constraints contain linearly arbitrary parameters h_1 , h_2 respectively or give zero⁶ $M^2 = m^2 + m'^2 = 0$.

It is easy to see that the additional parts do not possess the needed Hermitian conjugation properties

$$(l'_{ij})^+ \neq l'^+, \quad (g'_{12})^+ \neq g'_{21}. \quad (24)$$

The violation occurs in the (b_{ij}^+, b^+) sector of the auxiliary Fock space. Therefore to restore the Hermitian conjugation properties we change the scalar product in this sector of the Fock space

$$\langle \Psi_1 | \Psi_2 \rangle_{new} = \langle \Psi_1 | K | \Psi_2 \rangle \quad (25)$$

with some operator K to be defined. This operator must restore the Hermitian conjugation properties

$$\langle \Psi_1 | K l'_{ij} | \Psi_2 \rangle = \langle \Psi_2 | K l'^+_{ij} | \Psi_1 \rangle^*, \quad \langle \Psi_1 | K g'_{12} | \Psi_2 \rangle = \langle \Psi_2 | K g'_{21} | \Psi_1 \rangle^*. \quad (26)$$

The operator K can be written in the form (see e.g. [7] for a more detailed explanation)

$$K = Z^+ Z, \quad (27)$$

$$Z = \sum_{n_{ij}, n_G} (l_{11}^+)^{n_{11}} (l_{22}^+)^{n_{22}} (l_{12}^+)^{n_{12}} (g_{12}^+)^{n_G} |0\rangle_V \frac{1}{n_{11}! n_{22}! n_{12}! n_G!} \langle 0 | (b_{11})^{n_{11}} (b_{22})^{n_{22}} (b_{12})^{n_{12}} (b)^{n_G} \quad (28)$$

where the auxiliary state $|0\rangle_V$ satisfies the relations

$$l'_{11}|0\rangle_V = l'_{22}|0\rangle_V = l'_{12}|0\rangle_V = g'_{12}|0\rangle_V = 0, \quad {}_V\langle 0|0\rangle_V = 1 \quad (29)$$

$${}_V\langle 0|l_{11}^+ = {}_V\langle 0|l_{22}^+ = {}_V\langle 0|l_{12}^+ = {}_V\langle 0|g_{12}^+ = 0. \quad (30)$$

Thus in this section we construct the additional parts (13)–(20) which satisfied all the requirements.

⁶In what follows we forget about operator M^2 since its enlarged expression is zero.

4 Constructing BRST-BFV operator and Lagrangian

According to our method we should find the BRST-BFV operator. Since the algebra under consideration is a Lie algebra this operator can be constructed in the standard way⁷. First we introduce the ghost fields with the anticommutation relations

$$\{\eta_0, P_0\} = \{\eta_{12}^+, P_{12}\} = \{\eta_{12}, P_{12}^+\} = \{\eta_{12}^{G+}, P_{12}^G\} = \{\eta_{12}^G, P_{12}^{G+}\} = 1 \quad (31)$$

$$\{\eta_i^+, P_k\} = \{\eta_i, P_k^+\} = \{\eta_{ii}^+, P_{kk}\} = \{\eta_{ii}, P_{kk}^+\} = \{\eta_{ii}^G, P_{kk}^G\} = \delta_{ik} \quad (32)$$

with ghost numbers +1 for η 's and -1 for P 's. Then the the BRST-BFV operator has the form

$$\tilde{Q} = Q + [\eta_{11}^G(\sigma_1 + h_1) - (\eta_{11}^+ \eta_{11} + \eta_{12}^+ \eta_{12} + \eta_{12}^{G+} \eta_{12}^G) P_{11}^G + (1 \leftrightarrow 2)], \quad \tilde{Q}^2 = 0, \quad (33)$$

where, for the later use, we have decomposed \tilde{Q} in terms of η_{11}^G and η_{22}^G and their conjugate momenta. In (33) three operators have been introduced

$$\begin{aligned} Q = & \eta_0 L_0 + \sum_i (\eta_i^+ L_i + \eta_i L_i^+) + \sum_{i \leq j} (\eta_{ij}^+ L_{ij} + \eta_{ij} L_{ij}^+) + \eta_{12}^{G+} G_{12} + \eta_{12}^G G_{12}^+ \\ & + \left[-\eta_1^+ \eta_1 P_0 + \eta_1^+ (\eta_{11} P_1^+ + \frac{1}{2} \eta_{12} P_2^+) - \eta_1 (\eta_{11}^+ P_1 + \frac{1}{2} \eta_{12}^+ P_2) \right. \\ & + \eta_{12}^{G+} (\eta_1^+ P_2 - \eta_2 P_1^+ + 2\eta_{11}^+ P_{12} - 2\eta_{22} P_{12}^+ + \eta_{12}^+ P_{22} - \eta_{12} P_{11}^+) \\ & \left. - \frac{1}{2} (\eta_{22}^+ \eta_{12} + \eta_{12}^+ \eta_{11}) P_{12}^G + (1 \leftrightarrow 2) \right] \end{aligned} \quad (34)$$

and

$$\sigma_i + h_i = G_{ii} + (\eta_i^+ P_i - \eta_i P_i^+) + 2(\eta_{ii}^+ P_{ii} - \eta_{ii} P_{ii}^+) + (\eta_{12}^+ P_{12} - \eta_{12} P_{12}^+) \mp (\eta_{12}^{G+} P_{12}^G - \eta_{12}^G P_{12}^{G+}), \quad (35)$$

$$[\tilde{Q}, \sigma_i] = 0, \quad [Q, \sigma_i] = 0, \quad [\sigma_i, \sigma_j] = 0, \quad (36)$$

where in \mp the upper sign corresponds to $i = 1$ and the lower one corresponds to $i = 2$.

Further, we choose a representation of the Hilbert space given by the relations

$$(\eta_i, \eta_{ii}, \eta_{12}, \eta_{12}^G, P_0, P_i, P_{ii}, P_{12}, P_{12}^G, P_{ii}^G) |0\rangle = 0 \quad (37)$$

and suppose that the field vectors as well as the gauge parameters do not depend on η_{ii}^G

$$\begin{aligned} |\chi\rangle = & \sum_n (a_{1\mu_1}^+ \cdots a_{1\mu_{n_{a1}}}^+) (a_{2\nu_1}^+ \cdots a_{2\nu_{n_{a2}}}^+) \\ & \times (b_1^+)^{n_{b1}} (b_2^+)^{n_{b2}} (b_{11}^+)^{n_{b11}} (b_{22}^+)^{n_{b22}} (b_{12}^+)^{n_{b12}} (b^+)^{n_b} \\ & \times (\eta_0^+)^{n_{f0}} (\eta_1^+)^{n_{f1}} (P_1^+)^{n_{p1}} (\eta_2^+)^{n_{f2}} (P_2^+)^{n_{p2}} \\ & \times (\eta_{11}^+)^{n_{f11}} (P_{11}^+)^{n_{p11}} (\eta_{22}^+)^{n_{f22}} (P_{22}^+)^{n_{p22}} (\eta_{12}^+)^{n_{f12}} (P_{12}^+)^{n_{p12}} (\eta_{12}^{G+})^{n_{fg12}} (P_{12}^{G+})^{n_{pg12}} \\ & \times \chi(x)_{\mu_1 \cdots \mu_{n_{a1}} \nu_1 \cdots \nu_{n_{a2}} n_{p1} n_{p2} n_{p11} n_{p22} n_{p12} n_{f12}} |0\rangle. \end{aligned} \quad (38)$$

The sum in (38) is taken over n_{ai} , n_{bi} , n_{bij} , n_b running from 0 to infinity, and over the rest n 's from 0 to 1. Let us denote by $|\chi^k\rangle$ the state (38) with the ghost number $-k$, i.e. $gh(|\chi^k\rangle) = -k$. Thus the physical state having the ghost number zero is $|\chi^0\rangle$, the gauge parameters having the ghost number -1 is $|\chi^1\rangle$ and so on.

⁷Use of the BRST approach to higher spin field theory in AdS space leads to problem of constructing a BRST-BFV operator for non-linear algebras, see e.g. [12].

Since the vectors (38) do not depend on η_{ii}^G the equation for the physical state $\tilde{Q}|\chi^0\rangle = 0$ yields three equations

$$Q|\chi^0\rangle = 0, \quad (39)$$

$$(\sigma_i + h_i)|\chi^0\rangle = 0, \quad i = 1, 2. \quad (40)$$

Equations (39), (40) are compatible due to (36). Equations (40) present equations for possible values of h_i

$$h_i = -\left(n_i + \frac{d-7\pm 2}{2}\right), \quad n_1 = 0, \pm 1, \pm 2, \dots \quad n_2 = 0, 1, 2, \dots \quad (41)$$

Let us denote the eigenvectors of σ_i corresponding to the eigenvalues $n_i + \frac{d-7\pm 2}{2}$ as $|\chi\rangle_{n_1, n_2}$. Thus we may write

$$\sigma_i|\chi\rangle_{n_1, n_2} = \left(n_i + \frac{d-7\pm 2}{2}\right)|\chi\rangle_{n_1, n_2}. \quad (42)$$

Then one can show that in order to construct Lagrangian for the field corresponding to a definite Young tableau (1) the numbers n_i must be equal to the numbers of the boxes in the i -th row of the corresponding Young tableau, i.e. $n_i = s_i$. Thus the state $|\chi\rangle_{s_1 s_2}$ contains the physical field (1) and all its auxiliary fields. Let us fix some values of $n_i = s_i$. Then one should substitute h_i corresponding to the chosen n_i (41) into (33), (39). Thus the equation of motion (39) corresponding to the field with given spin (s_1, s_2) has the form

$$Q_{n_1 n_2}|\chi^0\rangle_{n_1 n_2} = 0. \quad (43)$$

Since the BRST-BFV operator \tilde{Q} is nilpotent (33) at any values of h_i we have a sequence of reducible gauge transformation⁸

$$\delta|\chi^0\rangle_{n_1, n_2} = Q_{n_1, n_2}|\chi^1\rangle_{n_1, n_2}, \quad \delta|\chi^1\rangle_{n_1, n_2} = Q_{n_1, n_2}|\chi^2\rangle_{n_1, n_2}, \quad (44)$$

$$\dots \quad \dots \quad \dots \quad (45)$$

$$\delta|\chi^5\rangle_{n_1, n_2} = Q_{n_1, n_2}|\chi^6\rangle_{n_1, n_2}, \quad \delta|\chi^6\rangle_{n_1, n_2} = 0. \quad (46)$$

One can show that Q_{n_1, n_2} is nilpotent when acting $|\chi\rangle_{n_1, n_2}$

$$Q_{n_1, n_2}^2|\chi\rangle_{n_1, n_2} \equiv 0. \quad (47)$$

Thus we have obtained equation of motion (43) of arbitrary spin gauge theory with mixed symmetry in any space-time dimension and its tower of reducible gauge transformations (44)–(46).

We next find a corresponding Lagrangian. Analogously to the bosonic one row case [7] one can show that Lagrangian for fixed spin (n_1, n_2) is defined up to an overall factor as follows

$$\mathcal{L}_{n_1, n_2} = \int d\eta_0 \, {}_{n_1, n_2} \langle \chi | K_{n_1, n_2} Q_{n_1, n_2} | \chi \rangle_{n_1, n_2} \quad (48)$$

where the standard scalar product for the creation and annihilation operators is assumed and the operator $K_{n_1 n_2}$ is the operator K (27) where the following substitution is done $h \rightarrow -(n_i + (d-7\pm 2)/2)$.

In the next section we show that the constraints (9) can indeed be reproduced from (43) up to gauge transformations (44)–(46).

⁸ Since there are six momentum ghosts P^+ 's having negative ghost number, which are Grassman odd operators, the lowest ghost number of $|\chi\rangle$ is -6 when $|\chi\rangle \sim P_1^+ P_2^+ P_{11}^+ P_{22}^+ P_{12}^+ P_{12}^{G+} |\text{no ghosts}\rangle$.

5 Reproduction of the initial constraints

Let us show that the equations of motion (2)–(5) [or equivalently (9)] can be obtained from (43) after partial gauge-fixing and removing the auxiliary fields by using a part of the equations of motion.

5.1 Gauge-fixing

Let us consider the field $|\chi^k\rangle$ at some fixed values of the spin (n_1, n_2) . In this section we will omit the subscripts associated with the eigenvalues of the σ_i operators (42). Then we extract dependence of Q (34) on zero ghosts η_0 and P_0

$$\begin{aligned}
Q &= \eta_0 L_0 - (\eta_1^+ \eta_1 + \eta_2^+ \eta_2) P_0 + \Delta Q \\
\Delta Q &= \sum_{i=1,2} (\eta_i^+ L_i + \eta_i L_i^+) + \sum_{(ij)=11,12,22} (\eta_{ij}^+ L_{ij} + \eta_{ij} L_{ij}^+) + \eta_{21}^G G_{12} + \eta_{12}^G G_{21} \\
&\quad + \eta_1^+ (\eta_{11} P_1^+ + \frac{1}{2} \eta_{12} P_2^+) - \eta_1 (\eta_{11}^+ P_1 + \frac{1}{2} \eta_{12}^+ P_2) \\
&\quad + \eta_2^+ (\eta_{22} P_2^+ + \frac{1}{2} \eta_{12} P_1^+) - \eta_2 (\eta_{22}^+ P_2 + \frac{1}{2} \eta_{12}^+ P_1) \\
&\quad + \eta_{12}^{G+} (\eta_1^+ P_2 - \eta_2 P_1^+ + 2\eta_{11}^+ P_{12} - 2\eta_{22} P_{12}^+ + \eta_{12}^+ P_{22} - \eta_{12} P_{11}^+) \\
&\quad + \eta_{12}^G (\eta_2^+ P_1 - \eta_1 P_2^+ + 2\eta_{22}^+ P_{12} - 2\eta_{11} P_{12}^+ + \eta_{12}^+ P_{11} - \eta_{12} P_{22}^+) \\
&\quad - \frac{1}{2} (\eta_{22}^+ \eta_{12} + \eta_{12}^+ \eta_{11}) P_{12}^G - \frac{1}{2} (\eta_{11}^+ \eta_{12} + \eta_{12}^+ \eta_{22}) P_{12}^{G+}
\end{aligned} \tag{49}$$

and do the same for the fields and gauge parameters

$$|\chi^k\rangle = |S^k\rangle + \eta_0 |A^k\rangle. \tag{51}$$

Then the gauge transformations and equations of motion (43)–(46) can be rewritten as follows

$$\delta |S^{k-1}\rangle = \Delta Q |S^k\rangle - (\eta_1^+ \eta_1 + \eta_2^+ \eta_2) |A^k\rangle \quad \delta |S^{-1}\rangle \equiv 0, \tag{52}$$

$$\delta |A^{k-1}\rangle = L_0 |S^k\rangle - \Delta Q |A^k\rangle \quad \delta |A^{-1}\rangle \equiv 0. \tag{53}$$

Let us consider the lowest level gauge transformation

$$\delta |S^5\rangle = \Delta Q |S^6\rangle, \quad \delta |A^5\rangle = L_0 |S^6\rangle, \tag{54}$$

where due to the ghost number restriction one has used that $|A^6\rangle \equiv 0$. Extracting explicitly dependence of the gauge parameters and of the operator ΔQ (50) on η_{11} , P_{11}^+ ghosts

$$|\chi^k\rangle = |\chi_0^k\rangle + P_{11}^+ |\chi_1^k\rangle, \quad \Delta Q = \Delta Q_1 + \eta_{11} T_1^+ + U_1 P_{11}^+, \tag{55}$$

where $|\chi_0^k\rangle$, $|\chi_1^k\rangle$, T_1^+ , U_1 do not depend on η_{11} , P_{11}^+ we get the gauge transformation of $|S_0^5\rangle$

$$\delta |S_0^5\rangle = T_1^+ |S_1^6\rangle. \tag{56}$$

Here we have used that $|S_0^6\rangle \equiv 0$ due to the ghost number restriction. Since $T_1^+ = b_{11}^+ + \dots$ we can remove dependence of $|S_0^5\rangle$ on b_{11}^+ using all the degrees of freedom of $|S_1^6\rangle$. Thus, after the gauge fixing at the lowest level of the gauge transformations we have conditions on $|S_0^5\rangle$

$$b_{11} |S_0^5\rangle = 0 \iff b_{11} P_{11}^+ |\chi^5\rangle = 0. \tag{57}$$

Let us turn to the next level of the gauge transformation. Extracting explicit dependence of the gauge parameters and ΔQ on η_{11} , P_{11}^+ and on η_{22} , P_{22}^+ and using similar arguments as at the previous level of the gauge transformation one can show that the gauge on $|\chi^4\rangle$

$$b_{11}P_{11}^+|\chi^4\rangle = 0, \quad b_{22}P_{22}^+P_{11}^+|\chi^4\rangle = 0. \quad (58)$$

can be imposed. To obtain these gauge conditions all degrees of freedom of the gauge parameters $|\chi^5\rangle$ restricted by (57) must be used.

Applying a similar procedure one can obtain step by step

$$b_{11}P_{11}^+|\chi^3\rangle = 0, \quad b_{22}P_{22}^+P_{11}^+|\chi^3\rangle = 0, \quad b_{12}P_{12}^+P_{22}^+P_{11}^+|\chi^3\rangle = 0. \quad (59)$$

Then

$$b_{11}P_{11}^+|\chi^2\rangle = 0, \quad b_{22}P_{22}^+P_{11}^+|\chi^2\rangle = 0, \quad b_{12}P_{12}^+P_{22}^+P_{11}^+|\chi^2\rangle = 0, \quad b_1P_1^+P_{12}^+P_{22}^+P_{11}^+|\chi^2\rangle = 0. \quad (60)$$

After this

$$b_{11}P_{11}^+|\chi^1\rangle = 0, \quad b_{12}P_{12}^+P_{22}^+P_{11}^+|\chi^1\rangle = 0, \quad b_2P_2^+P_1^+P_{12}^+P_{22}^+P_{11}^+|\chi^1\rangle = 0, \quad (61)$$

$$b_{22}P_{22}^+P_{11}^+|\chi^1\rangle = 0, \quad b_1P_1^+P_{12}^+P_{22}^+P_{11}^+|\chi^1\rangle = 0. \quad (62)$$

And finally we obtain gauge conditions on the field $|\chi^0\rangle$

$$b_{11}P_{11}^+|\chi^0\rangle = 0, \quad b_{12}P_{12}^+P_{22}^+P_{11}^+|\chi^0\rangle = 0, \quad b_2P_2^+P_1^+P_{12}^+P_{22}^+P_{11}^+|\chi^0\rangle = 0, \quad (63)$$

$$b_{22}P_{22}^+P_{11}^+|\chi^0\rangle = 0, \quad b_1P_1^+P_{12}^+P_{22}^+P_{11}^+|\chi^0\rangle = 0, \quad bP_{12}^{G+}P_2^+P_1^+P_{12}^+P_{22}^+P_{11}^+|\chi^0\rangle = 0. \quad (64)$$

Let us now turn to removing the auxiliary fields using the equations of motion.

5.2 Removing auxiliary fields by means of equations of motion

First we decompose the fields $|S^0\rangle$ as follows

$$|S^0\rangle = |S_0^0\rangle + P_{11}^+|S_1^0\rangle, \quad |S_{000}^0\rangle = |S_{0000}^0\rangle + P_1^+|S_{0001}^0\rangle, \quad (65)$$

$$|S_0^0\rangle = |S_{00}^0\rangle + P_{22}^+|S_{01}^0\rangle, \quad |S_{0000}^0\rangle = |S_{00000}^0\rangle + P_2^+|S_{00001}^0\rangle, \quad (66)$$

$$|S_{00}^0\rangle = |S_{000}^0\rangle + P_{12}^+|S_{001}^0\rangle, \quad |S_{00000}^0\rangle = |S_{000000}^0\rangle + P_{12}^{G+}|S_{000001}^0\rangle \quad (67)$$

and do the same for $|A^0\rangle$

$$|A^0\rangle = P_{12}^{G+}|A_{000001}^0\rangle + P_2^+|A_{00001}^0\rangle + P_1^+|A_{0001}^0\rangle + P_{12}^+|A_{001}^0\rangle + P_{22}^+|A_{01}^0\rangle + P_{11}^+|A_1^0\rangle, \quad (68)$$

where the term independent of the ghost momenta is absent due to the ghost number restriction. We note that due to $gh(|S^0\rangle) = 0$, $|S_{000000}^0\rangle$ can't depend on the ghost coordinates and as a consequence of the gauge conditions (63), (64) $|S_{000000}^0\rangle = |\Phi\rangle$, with $|\Phi\rangle$ being the physical field (7).

Then analogously to the fields we extract in ΔQ (50) first dependence on η_{11} , P_{11}^+ , next dependence on η_{22} , P_{22}^+ , and further on η_{12} , P_{12}^+ , on η_1 , P_1^+ , on η_2 , P_2^+ , and on η_{12}^G , P_{12}^{G+} respectively.

Substituting these decompositions into the equation of motion

$$l_0|S^0\rangle - \Delta Q|A^0\rangle = 0 \quad (69)$$

and using the gauge conditions (63), (64) one can show that first $|A_{000001}^0\rangle = 0$, then $|A_{00001}^0\rangle = 0$, and so on till $|A_1^0\rangle = 0$ which means that

$$l_0|S^0\rangle = 0, \quad |A^0\rangle = 0. \quad (70)$$

Analogously we consider the second equation of motion

$$\Delta Q|S^0\rangle = 0, \quad (71)$$

where $|A^0\rangle = 0$ has been used. After the same decomposition we conclude one after another that

$$|S_{000001}^0\rangle = |S_{00001}^0\rangle = |S_{0001}^0\rangle = |S_{001}^0\rangle = |S_{01}^k\rangle = |S_1^k\rangle = 0. \quad (72)$$

Eqs. (70) and (72) mean that all the auxiliary fields vanish and as a result we have $|\chi^0\rangle = |\Phi\rangle$ and the equations of motion (9) are fulfilled.

Let us now turn to examples.

6 Examples

6.1 Spin-($s, 0$) totally symmetric field

Let us consider the totally symmetric field corresponding to spin- $(s, 0)$. In this case we expect that our result will be reduced to that considered in [7], where the totally symmetric massive bosonic fields were considered. According to our procedure we have $n_1 = s$, $n_2 = 0$. One can show that if $n_2 = 0$ then in (38) all the components related with the second row in the Young tableau must be equal to zero, i.e.

$$n_{a2} = n_{b2} = n_{b22} = n_{b21} = n_b = n_{f2} = n_{p2} = n_{f22} = n_{p22} = n_{f12} = n_{p12} = n_{fg12} = n_{pg12} = 0. \quad (73)$$

Thus the state vector is reduced to

$$\begin{aligned} |\chi\rangle &= \sum_n (a_{1\mu_1}^+ \cdots a_{1\mu_{n_{a1}}}^+) (b_1^+)^{n_{b1}} (b_{11}^+)^{n_{b11}} (\eta_0^+)^{n_{f0}} (\eta_1^+)^{n_{f1}} (P_1^+)^{n_{p1}} (\eta_{11}^+)^{n_{f11}} (P_{11}^+)^{n_{p11}} \\ &\times \chi(x)_{\mu_1 \cdots \mu_{n_{a1}} 0 n_{b1} 0 n_{b11} 000 n_{f0} n_{f1} 0 n_{f11} 000}^{n_{b1} 0 n_{b11} 000 n_{f0} n_{f1} 0 n_{f11} 000} |0\rangle, \end{aligned} \quad (74)$$

which corresponds to that in [7]. Then one can easily show that equations (43), (44), (48) with $|\chi\rangle$ as in (74) reproduce the same relations as those in [7].

6.2 Rank-2 antisymmetric tensor field

The next example is the simplest mixed symmetric case, that is, spin- $(1, 1)$ rank-2 totally antisymmetric tensor field. In this example we denote all the gauge parameters by letters with primes and all the gauge parameters of the second level of the gauge transformations by letters with two primes.

6.2.1 Lagrangian

Let us decompose the state vector (38) having ghost number zero⁹ and obeying (42) at $n_1 = n_2 = 1$ first in the ghost fields and then in auxiliary creation operators b_{11}^+ , b_{22}^+ , b_{12}^+ , b^+

$$\begin{aligned} |\chi^0\rangle_{1,1} &= |B\rangle_{1,1} + b^+ |T_1\rangle_{2,0} + b_{12}^+ |\phi_1\rangle_{0,0} + b_{11}^+ b^+ |\phi_2\rangle_{0,0} \\ &\quad + P_1^+ \eta_1^+ b^+ |\phi_3\rangle_{0,0} + P_1^+ \eta_2^+ |\phi_4\rangle_{0,0} + P_2^+ \eta_1^+ |\phi_5\rangle_{0,0} \\ &\quad + \eta_{12}^{G+} P_1^+ |A_1\rangle_{1,0} + \eta_{12}^{G+} P_{11}^+ |\phi_6\rangle_{0,0} + P_{12}^{G+} \eta_1^+ |A_2\rangle_{1,0} + P_{12}^{G+} \eta_{11}^+ |\phi_7\rangle_{0,0} \\ &\quad + \eta_0 \left(P_1^+ (|H\rangle_{0,1} + b^+ |A_3\rangle_{1,0}) + P_2^+ |A_4\rangle_{1,0} + P_{11}^+ b^+ |\phi_8\rangle_{0,0} + P_{12}^+ |\phi_{10}\rangle_{0,0} \right. \\ &\quad \left. + P_{12}^{G+} (|T_2\rangle_{2,0} + b_{11}^+ |\phi_9\rangle_{0,0}) + P_{12}^{G+} P_1^+ \eta_1^+ |\phi_{11}\rangle_{0,0} \right), \end{aligned} \quad (75)$$

⁹The states (38) in spin- $(1, 1)$ case have the lowest ghost number -2 due to restriction (42) at $n_1 = n_2 = 1$.

where states $|\cdot\rangle$ still depend on $a_{1,\mu}^+$, $a_{2,\mu}^+$, b_1^+ , b_2^+ and are expanded as follows

$$|B\rangle_{1,1} = -a_1^{+\mu} a_2^{+\nu} |0\rangle B_{\mu\nu}(x) - ib_1^+ a_2^{+\mu} |0\rangle H_{(2)\mu}(x) - ib_2^+ a_1^{+\mu} |0\rangle A_{(7)\mu}(x) + b_1^+ b_2^+ |0\rangle \phi_{(19)}(x) \quad (76)$$

$$|T_i\rangle_{2,0} = -a_1^{+\mu} a_1^{+\nu} |0\rangle T_{(i)(\mu\nu)}(x) - ib_1^+ a_1^{+\mu} |0\rangle A_{(4+i)\mu}(x) + (b_1^+)^2 |0\rangle \phi_{(16+i)}(x) \quad (77)$$

$$|H\rangle_{0,1} = -ia_2^{+\mu} |0\rangle H_\mu(x) + b_2^+ |0\rangle \phi_{(16)}(x) \quad (78)$$

$$|A_i\rangle_{1,0} = -ia_1^{+\mu} |0\rangle A_{(i)\mu}(x) + b_1^+ |0\rangle \phi_{(11+i)}(x) \quad (79)$$

$$|\phi_i\rangle_{0,0} = |0\rangle \phi_{(i)}(x) \quad (80)$$

Then the relation (48) gives the following Lagrangian

$$\begin{aligned} \mathcal{L}_{(1,1)} = & B_{\mu\nu}(\partial^2 + m^2)B^{\mu\nu} - 4T_{(2)(\mu\nu)}B^{\mu\nu} - 4T_{(1)(\mu\nu)}(\partial^2 + m^2)T_{(1)}^{\mu\nu} + 8T_{(1)(\mu\nu)}T_{(2)}^{\mu\nu} \\ & + 2A_{(4)\mu}\partial_\nu B^{\mu\nu} + 2H_\nu\partial_\mu B^{\mu\nu} - 4A_{(1)\mu}\partial_\nu T_{(2)}^{\mu\nu} - 8A_{(3)\mu}\partial_\nu T_{(1)}^{\mu\nu} + \phi_{(10)}B_\mu^\mu - 2\phi_{(6)}T_{\mu(2)}^\mu \\ & - 4\phi_{(8)}T_{\mu(1)}^\mu - 2A_{(1)\mu}(\partial^2 + m^2)A_{(2)}^\mu + 2A_{(5)\mu}(\partial^2 + m^2)A_{(5)}^\mu - A_{(7)\mu}(\partial^2 + m^2)A_{(7)}^\mu \\ & - H_{(2)\mu}(\partial^2 + m^2)H_{(2)}^\mu - 2mA_{(1)\mu}A_{(6)}^\mu - 4A_{(2)\mu}A_{(3)}^\mu + 2A_{(2)\mu}A_{(4)}^\mu + 2A_{(2)\mu}H^\mu \\ & + 2A_{(3)\mu}A_{(3)}^\mu - 4mA_{(3)\mu}A_{(5)}^\mu - A_{(4)\mu}A_{(4)}^\mu + 2mA_{(4)\mu}A_{(7)}^\mu - 4A_{(5)\mu}A_{(6)}^\mu + 2A_{(6)\mu}A_{(7)}^\mu \\ & + 2A_{(6)\mu}H_{(2)}^\mu - H_\mu H^\mu + 2mH_\mu H_{(2)}^\mu + 2\phi_{(11)}\partial_\mu A_{(1)}^\mu + 4\phi_{(3)}\partial_\mu A_{(3)}^\mu - 2\phi_{(4)}\partial_\mu A_{(4)}^\mu \\ & + 4\phi_{(14)}\partial_\mu A_{(5)}^\mu + 2\phi_{(12)}\partial_\mu A_{(6)}^\mu - 2\phi_{(16)}\partial_\mu A_{(7)}^\mu - 2\phi_{(5)}\partial_\mu H^\mu - 2\phi_{(15)}\partial_\mu H_{(2)}^\mu \\ & + \frac{5-d}{4}\phi_{(1)}(\partial^2 + m^2)\phi_{(1)} - 2\phi_{(1)}(\partial^2 + m^2)\phi_{(2)} + (d-1)\phi_{(2)}(\partial^2 + m^2)\phi_{(2)} \\ & - 2\phi_{(1)}(\partial^2 + m^2)\phi_{(2)} + (d-1)\phi_{(2)}(\partial^2 + m^2)\phi_{(2)} + 2\phi_{(3)}(\partial^2 + m^2)\phi_{(3)} \\ & - 2\phi_{(4)}(\partial^2 + m^2)\phi_{(5)} + 2\phi_{(6)}(\partial^2 + m^2)\phi_{(7)} + 2\phi_{(12)}(\partial^2 + m^2)\phi_{(13)} \\ & - 4\phi_{(17)}(\partial^2 + m^2)\phi_{(17)} + \phi_{(19)}(\partial^2 + m^2)\phi_{(19)} + 2\phi_{(1)}\phi_{(8)} + (d-3)\phi_{(1)}\phi_{(9)} \\ & + \frac{d-5}{2}\phi_{(1)}\phi_{(10)} + 2(1-d)\phi_{(2)}\phi_{(8)} + 2(3-d)\phi_{(2)}\phi_{(9)} + 2\phi_{(2)}\phi_{(10)} - 4\phi_{(3)}\phi_{(8)} \\ & - 4\phi_{(3)}\phi_{(11)} + 4m\phi_{(3)}\phi_{(14)} + \phi_{(4)}\phi_{(10)} + 2\phi_{(4)}\phi_{(11)} - 2m\phi_{(4)}\phi_{(15)} + \phi_{(5)}\phi_{(10)} \\ & + 2\phi_{(5)}\phi_{(11)} - 2m\phi_{(5)}\phi_{(16)} + (3-d)\phi_{(6)}\phi_{(9)} + \phi_{(6)}\phi_{(10)} - 2\phi_{(6)}\phi_{(11)} - 2\phi_{(6)}\phi_{(18)} \\ & + 4\phi_{(7)}\phi_{(8)} - 2\phi_{(7)}\phi_{(10)} - 4\phi_{(8)}\phi_{(17)} + \phi_{(10)}\phi_{(19)} + 2m\phi_{(11)}\phi_{(12)} + 4m\phi_{(12)}\phi_{(18)} \\ & + 4\phi_{(13)}\phi_{(14)} - 2\phi_{(13)}\phi_{(15)} - 2\phi_{(13)}\phi_{(16)} - 2\phi_{(14)}\phi_{(14)} + 8m\phi_{(14)}\phi_{(17)} + \phi_{(15)}\phi_{(15)} \\ & - 2m\phi_{(15)}\phi_{(19)} + \phi_{(16)}\phi_{(16)} - 2m\phi_{(16)}\phi_{(19)} + 8\phi_{(17)}\phi_{(18)} - 4\phi_{(18)}\phi_{(19)}. \end{aligned} \quad (81)$$

Let us turn to the gauge transformations.

6.2.2 The gauge transformations for the fields

Now we decompose the vector (38) obeying the (42) at $n_1 = n_2 = 1$ and having ghost number -1 as follows

$$\begin{aligned} |\chi^1\rangle_{1,1} = & P_1^+ (|H'\rangle_{0,1} + b^+ |A'_1\rangle_{1,0}) + P_2^+ |A'_2\rangle_{1,0} + P_{11}^+ b^+ |\phi'_1\rangle_{0,0} + P_{12}^+ |\phi'_2\rangle_{0,0} \\ & + P_{12}^{G+} (|T'_1\rangle_{2,0} + b_{11}^+ |\phi'_3\rangle_{0,0}) + P_{12}^{G+} P_1^+ \eta_1^+ |\phi'_4\rangle_{0,0} \\ & + \eta_0 \left(P_1^+ P_2^+ |\phi'_5\rangle_{0,0} + P_1^+ P_{12}^{G+} |A'_3\rangle_{1,0} + P_{11}^+ P_{12}^{G+} |\phi'_6\rangle_{0,0} \right), \end{aligned} \quad (82)$$

where

$$|T'\rangle_{2,0} = -a_1^{+\mu} a_1^{+\nu} |0\rangle T'_{\mu\nu}(x) - ib_1^+ a_1^{+\mu} |0\rangle A'_{(4)\mu}(x) + (b_1^+)^2 |0\rangle \phi'_{(11)}(x) \quad (83)$$

$$|H'\rangle_{0,1} = -ia_2^{+\mu} |0\rangle H'_\mu(x) + b_2^+ |0\rangle \phi'_{(10)}(x) \quad (84)$$

$$|A'_i\rangle_{1,0} = -ia_1^{+\mu} |0\rangle A'_{(i)\mu}(x) + b_1^+ |0\rangle \phi'_{(6+i)}(x) \quad (85)$$

$$|\phi'_i\rangle_{0,0} = |0\rangle \phi'_{(i)}(x). \quad (86)$$

Substituting (75), (82) in the left relations (44) we obtain gauge transformations for the fields

$$\delta B_{\mu\nu} = 2T'_{\mu\nu} + \partial_\mu H'_\nu + \partial_\nu A'_{(2)\mu} - \frac{\eta_{\mu\nu}}{2}\phi'_{(2)} \quad (87)$$

$$\delta T_{(1)\mu\nu} = T'_{\mu\nu} + \partial_{(\mu} A'_{(1)\nu)} - \frac{\eta_{\mu\nu}}{2}\phi'_{(1)} \quad (88)$$

$$\delta T_{(2)\mu\nu} = (\partial^2 + m^2)T'_{\mu\nu} - \partial_{(\mu} A'_{(3)\nu)} + \frac{\eta_{\mu\nu}}{2}\phi'_{(6)} \quad (89)$$

$$\delta A_{(1)\mu} = -2A'_{(1)\mu} + A'_{(2)\mu} + H'_\mu \quad (90)$$

$$\delta A_{(2)\mu} = -2\partial^\nu T'_{\mu\nu} - mA'_{(4)\mu} - \partial_\mu \phi'_{(4)} + A'_{(3)\mu} \quad (91)$$

$$\delta A_{(3)\mu} = (\partial^2 + m^2)A'_{(1)\mu} + A'_{(3)\mu} \quad (92)$$

$$\delta A_{(4)\mu} = -\partial_\mu \phi'_{(5)} + (\partial^2 + m^2)A'_{(2)\mu} + A'_{(3)\mu} \quad (93)$$

$$\delta A_{(5)\mu} = mA'_{(1)\mu} + \partial_\mu \phi'_{(7)} + A'_{(4)\mu} \quad (94)$$

$$\delta A_{(6)\mu} = -mA'_{(3)\mu} - \partial_\mu \phi'_{(9)} + (\partial^2 + m^2)A'_{(4)\mu} \quad (95)$$

$$\delta A_{(7)\mu} = mA'_{(2)\mu} + \partial_\mu \phi'_{(10)} + A'_{(4)\mu} \quad (96)$$

$$\delta H_\mu = \partial_\mu \phi'_{(5)} + (\partial^2 + m^2)H'_\mu + A'_{(3)\mu} \quad (97)$$

$$\delta H_{(2)\mu} = mH'_\mu + \partial_\mu \phi'_{(8)} + A'_{(4)\mu} \quad (98)$$

$$\delta \phi_{(1)} = \phi'_{(2)} + 2\phi'_{(3)} \quad (99)$$

$$\delta \phi_{(2)} = \phi'_{(1)} + \phi'_{(3)} \quad (100)$$

$$\delta \phi_{(3)} = -\partial^\mu A'_{(1)\mu} - m\phi'_{(7)} + \phi'_{(1)} + \phi'_{(4)} \quad (101)$$

$$\delta \phi_{(4)} = -\partial^\mu H'_\mu - m\phi'_{(10)} + \frac{1}{2}\phi'_{(2)} + \phi'_{(4)} - \phi'_{(5)} \quad (102)$$

$$\delta \phi_{(5)} = -\partial^\mu A'_{(2)\mu} - m\phi'_{(8)} + \frac{1}{2}\phi'_{(2)} + \phi'_{(4)} + \phi'_{(5)} \quad (103)$$

$$\delta \phi_{(6)} = -2\phi'_{(1)} + \phi'_{(2)} \quad (104)$$

$$\delta \phi_{(7)} = \frac{d-3}{2}\phi'_{(3)} + T'^\mu_\mu - \frac{1}{2}\phi'_{(2)} + \phi'_{(4)} + \phi'_{(11)} \quad (105)$$

$$\delta \phi_{(8)} = (\partial^2 + m^2)\phi'_{(1)} + \phi'_{(6)} \quad (106)$$

$$\delta \phi_{(9)} = (\partial^2 + m^2)\phi'_{(3)} - \phi'_{(6)} \quad (107)$$

$$\delta \phi_{(10)} = (\partial^2 + m^2)\phi'_{(2)} + 2\phi'_{(6)} \quad (108)$$

$$\delta \phi_{(11)} = \partial^\mu A'_{(3)\mu} + m\phi'_{(9)} + (\partial^2 + m^2)\phi'_{(4)} - \phi'_{(6)} \quad (109)$$

$$\delta \phi_{(12)} = -2\phi'_{(7)} + \phi'_{(8)} + \phi'_{(10)} \quad (110)$$

$$\delta \phi_{(13)} = -\partial^\mu A'_{(4)\mu} - m\phi'_{(4)} - 2m\phi'_{(11)} + \phi'_{(9)} \quad (111)$$

$$\delta \phi_{(14)} = (\partial^2 + m^2)\phi'_{(7)} + \phi'_{(9)} \quad (112)$$

$$\delta \phi_{(15)} = -m\phi'_{(5)} + (\partial^2 + m^2)\phi'_{(8)} + \phi'_{(9)} \quad (113)$$

$$\delta \phi_{(16)} = m\phi'_{(5)} + (\partial^2 + m^2)\phi'_{(10)} + \phi'_{(9)} \quad (114)$$

$$\delta \phi_{(17)} = m\phi'_{(7)} - \frac{1}{2}\phi'_{(1)} + \phi'_{(11)} \quad (115)$$

$$\delta \phi_{(18)} = -m\phi'_{(9)} + (\partial^2 + m^2)\phi'_{(11)} + \frac{1}{2}\phi'_{(6)} \quad (116)$$

$$\delta \phi_{(19)} = m\phi'_{(8)} + m\phi'_{(10)} - \frac{1}{2}\phi'_{(2)} + 2\phi'_{(11)} \quad (117)$$

Let us turn to the gauge transformation for the gauge parameters.

6.2.3 The gauge transformations for the gauge parameters

Let us decompose the vector (38) obeying the (42) at $n_1 = n_2 = 1$ and having ghost number -2 as follows

$$|\chi^2\rangle_{1,1} = P_1^+ P_2^+ |\phi_1''\rangle_{0,0} + P_1^+ P_{12}^{G+} |A''\rangle_{1,0} + P_{11}^+ P_{12}^{G+} |\phi_2''\rangle_{0,0}, \quad (118)$$

where

$$|A''\rangle_{1,0} = -ia_1^{+\mu} |0\rangle A''_\mu(x) + b_1^+ |0\rangle \phi_{(3)}''(x), \quad |\phi_i''\rangle_{0,0} = |0\rangle \phi_{(i)}''(x). \quad (119)$$

Substituting (82), (118) in the right relations (44) one gets the gauge transformations for the gauge parameters

$$\delta T'_{\mu\nu} = \partial_{(\mu} A''_{\nu)} - \frac{1}{2} \eta_{\mu\nu} \phi_{(2)}'', \quad \delta A'_{(1)\mu} = -A''_\mu, \quad (120)$$

$$\delta A'_{(2)\mu} = \partial_\mu \phi_{(1)}'' - A''_\mu, \quad \delta A'_{(3)\mu} = (\partial^2 + m^2) A''_\mu, \quad (121)$$

$$\delta A'_{(4)\mu} = m A''_\mu + \partial_\mu \phi_{(3)}'', \quad \delta H'_\mu = -\partial_\mu \phi_{(1)}'' - A''_\mu, \quad (122)$$

$$\delta \phi'_{(1)} = -\phi_{(2)}'', \quad \delta \phi'_{(2)} = -2\phi_{(2)}'', \quad (123)$$

$$\delta \phi'_{(3)} = \phi_{(2)}'', \quad \delta \phi'_{(4)} = -\partial^\mu A''_\mu - m \phi_{(3)}'' + \phi_{(2)}'', \quad (124)$$

$$\delta \phi'_{(5)} = (\partial^2 + m^2) \phi_{(1)}'', \quad \delta \phi'_{(6)} = (\partial^2 + m^2) \phi_{(2)}'', \quad (125)$$

$$\delta \phi'_{(7)} = -\phi_{(3)}'', \quad \delta \phi'_{(8)} = m \phi_{(1)}'' - \phi_{(3)}'', \quad (126)$$

$$\delta \phi'_{(9)} = (\partial^2 + m^2) \phi_{(3)}'', \quad \delta \phi'_{(10)} = -m \phi_{(1)}'' - \phi_{(3)}'', \quad (127)$$

$$\delta \phi'_{(11)} = m \phi_{(3)}'' - \frac{1}{2} \phi_{(2)}'', \quad (128)$$

6.2.4 Gauge fixing and partial use of equations of motion

Let us fix gauge symmetry completely by the gauge fixing conditions (63) and (64) obtained at general consideration. It is easy to see that following fields will be eliminated

$$\phi_{(1)}, \phi_{(2)}, \phi_{(9)}, \phi_{(13)}, \phi_{(15)}, \phi_{(17)}, \phi_{(18)}, \phi_{(19)}, A_{(5)}, A_{(6)}, A_{(7)}, H_{(2)}, T_{(1)} \longrightarrow 0. \quad (129)$$

Then using equations of motion for all fields but antisymmetric part of the basic field $B_{[\mu\nu]}$, one sees that only $B_{[\mu\nu]}$ remains and Lagrangian for it, up to total derivative, is

$$\mathcal{L}_{B_{[\mu\nu]}} = B_{[\mu\nu]} (\partial^2 + m^2) B^{[\mu\nu]} + 2(\partial_\mu B^{[\mu\lambda]})(\partial^\nu B_{[\nu\lambda]}) \quad (130)$$

$$= -\frac{1}{3} G_{\mu\nu\lambda} G^{\mu\nu\lambda} + m^2 B_{[\mu\nu]} B^{[\mu\nu]} + \text{total derivative} \quad (131)$$

where we have defined field strength $G_{\mu\nu\lambda}$ of $B_{[\mu\nu]}$

$$G_{\mu\nu\lambda} := \partial_\lambda B_{[\mu\nu]} + \partial_\mu B_{[\nu\lambda]} + \partial_\nu B_{[\lambda\mu]}. \quad (132)$$

Thus we have obtained the gauge-invariant Lagrangian (81) for massive rank-2 antisymmetric tensor field containing the complete set of auxiliary fields and gauge parameters.

7 Summary and discussion

We have developed the general gauge invariant approach to Lagrangian construction describing dynamics of massive bosonic higher spin fields with index symmetry corresponding to two-row

Young tableau (1) in flat space-time of arbitrary dimension. The obtained field model is a reducible gauge theory. The final Lagrangian and gauge transformations are given by (48) and (44)–(46) respectively. The Lagrangian automatically contains the appropriate set of the Stückelberg fields providing the gauge invariance of massive theory.

Generalization of this construction to the fields corresponding to k -row Young tableau is straightforward. In this case one should introduce K pairs of creation and annihilation operators (6), where $i, j = 1, 2, \dots, k$. Constraint algebra contains the generators $l_0, l_i, l_i^+, l_{ij}, l_{ij}^+, g_{ij}$ analogous to the case $k = 2$. The new representation can be obtained using the procedure developed in [7]. BRST-BFV operators \tilde{Q} and Q are defined straightforwardly and there will be k spin number operators σ_i which commute with Q . The eigenvalues of these operators are related with the spin of the field like in the two-row case. Then Lagrangian and gauge transformations are constructed analogously to the case $k = 2$ which was studied in the given paper.

Acknowledgements

H.T. is thankful to high energy theory groups of CQeST, Hiroshima U. and KEK for discussions and kind hospitality. The work of I.L.B and V.A.K was partially supported by the INTAS grant, project INTAS-05-7928, the RFBR grant, project No. 06-02-16346 and grant for LRSS, project No. 4489.2006.2. Work of I.L.B was supported in part by the DFG grant, project No. 436 RUS 113/669/0-3 and joint RFBR-DFG grant, project No. 06-02-04012. Work of V.A.K was partially supported by the joint DAAD-Mikhail Lomonosov Programm (Referat 325, Kennziffer A/06/16774).

References

- [1] S. Ouvry and J. Stern, Phys. Lett. B177 (1986) 335–340; J.M.F. Labastida, T.R. Morris, Phys. Lett. B180 (1986) 101–106; A.K.H. Bengtsson, Phys. Lett. B182 (1986) 321–325; J.M.F. Labastida, Phys. Lett. B186 (1987) 365–369; J.M.F. Labastida, Nucl. Phys. B322 (1989) 185–209.
- [2] L. Brink, R.R. Metsaev, M.A. Vasiliev, Nucl. Phys. B586 (2000) 183–205, hep-th/0005136.
- [3] K.B. Alkalaev, O.V. Shaynkman, M.A. Vasiliev, Nuc.Phys. B692 (2004) 363–393, hep-th/0311164. N. Boulanger, I. Kirsch, Phys. Rev. D73 (2006) 124023, hep-th/0602225.
- [4] N. Boulanger, I. Kirsch, Phys. Rev. D73 (2006) 124023, hep-th/0602225.
- [5] Yu. M. Zinoviev, On massive mixed symmetry tensor fields in Minkowski space and (A)dS, hep-th/0211233; First order formalism for mixed symmetry tensor fields, hep-th/0304067; First order formalism for massive mixed symmetry tensor fields in Minkowski and (A)dS spaces, hep-th/0306292; On dual formulations of massive tensor fields, hep-th/0504081; R.R. Metsaev, Class.Quant.Grav. 22 (2005) 2777–2796, hep-th/0412311.
- [6] A. Pashnev, M. Tsulaia, Mod.Phys.Lett. A13 (1998) 1853–1864, hep-th/9803207; I.L. Buchbinder, A. Pashnev, M. Tsulaia, Phys.Lett. B523 (2001) 338–346, hep-th/0109067; I.L. Buchbinder, A. Pashnev, M. Tsulaia, Massless higher spin fields in the AdS background and BRST constructions for nonlinear algebras, hep-th/0206026; A. Sagnotti, M. Tsulaia, Nucl. Phys. B682 (2004) 83–116, hep-th/0311257; X. Bekaert, I.L. Buchbinder, A. Pashnev, M. Tsulaia, Class.Quant.Grav. 21 (2004) S1457–S1464, hep-th/0410215; I.L. Buchbinder,

- V.A. Krykhtin, A. Pashnev, Nucl. Phys. B711 (2005) 367–391, hep-th/0410215; A. Fotopoulos, K.L. Panigrahi, M. Tsulaia, Phys. Rev. D74 (2006) 085029; I.L. Buchbinder, A.V. Galajinsky, V.A. Krykhtin, Quartet unconstrained formulation for massless higher spin fields, hep-th/0702161.
- [7] I.L. Buchbinder, V.A. Krykhtin, Nucl. Phys. B727 (2005) 536–563, hep-th/0505092; I.L. Buchbinder, V.A. Krykhtin, BRST approach to higher spin field theories, hep-th/0511276; I.L. Buchbinder, V.A. Krykhtin, L.L. Ryskina, H. Takata, Phys. Lett. B641 (2006) 386–392, hep-th/0603212; I.L. Buchbinder, V.A. Krykhtin, P.M. Lavrov, Nucl.Phys. B762 (2007) 344–376, hep-th/0608005; I.L. Buchbinder, V.A. Krykhtin, A.A. Reshetnyak, BRST approach to Lagrangian construction for fermionic higher spin fields in AdS space, hep-th/0703049.
 - [8] C. Burdik, A.Pashnev, M. Tsulaia, Mod.Phys.Lett. A16 (2001) 731–746, hep-th/0101201; P.Yu. Moshin, A.A. Reshetnyak, BRST approach to Lagrangian formulation for mixed-symmetry fermionic higher-spin fields, hep-th/0707.0386.
 - [9] A.K.H. Bengtsson, Class. Quant. Grav. 5 (1988) 437; L. Cappiello, M. Knecht, S. Ouvry, and J. Stern, Ann. Phys. 193 (1989) 10–39; F. Fougere, M. Knecht and J. Stern, Algebraic construction of higher spin interaction vertexes, preprint LAPP-TH-338/91; I.L. Buchbinder, A.Fotopoulos, A.C. Petkou, M. Tsulaia, Phys.Rev. D74 (2006) 105018, hep-th/0609082; A.Fotopoulos, M. Tsulaia, Interacting Higher Spins and the High Energy Limit of the Bosonic String, hep-th/0705.2939.
 - [10] K. Hallowell, A. Waldron, Supersymmetric Quantum Mechanics and Super-Lichnerowicz Algebras, hep-th/0702033.
 - [11] C. Burdik J. Phys A: Math. Gen 18 (1985) 3101; C. Burdik, A. Pashnev, M. Tsulaia, Mod. Phys. Lett. A15 (2000) 281–292, hep-th/0001195; C. Burdik, O. Navratil, A. Pashnev, On the Fock Space Realizations of Nonlinear Algebras Describing the High Spin Fields in AdS Spaces, hep-th/0206027.
 - [12] I.L. Buchbinder, P.M. Lavrov, Classical BRST charge for nonlinear algebras, hep-th/0701243.